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Research of Appearance and Propagation of Higher Harmonics of Acoustic Signals in the Nonlinear Media

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Abstract
This article describes mathematical modeling formation of higher-order acoustic harmonics in the nonlinear medium. Two problems are considered: propagation of the higher harmonics in the case of the plane wave (amplitude was deduced from nonlinear Earnshaw's equation) and the two-dimensional ultrasonic beam (KZK equation was used for deducing). Simulating change of amplitudes of the higher harmonics is carried out in biological subjects with distribution of a nonlinear parameter that is amenable to the Gaussian law. The results of numerical calculations of the dependences of the change of amplitudes of the higher order harmonics on a pattern of distribution of the nonlinear parameter in biological tissues are presented. Dependence which had been obtained enabled to proceed the inverse problem: imaging internal structures of biological objects using a nonlinear parameter as an informative indicator reflecting the properties of the medium.
Key words: acoustic tomography, nonlinear parameter, biological tissue, highest harmonic.

INTRODUCTION

Research of nonlinear properties of biological tissue enables the use of nonlinear parameter for the purposes of imaging internal structures of the human body [1; 2]. The research results of healthy and pathological tissues of pig's liver showed considerable changes of the nonlinear parameter in them from 9 to 20% in comparison with linear sound propagation characteristics: velocity from 2 to 3.8%, density about 1% [3]. This allowed to conclude that the nonlinear parameter was much more sensitive to changes in the status of biological tissue. Comparing the deviation of the nonlinearity parameter and linear characteristics, it can be assumed that its use as a diagnostic parameter for systems imaging has significant advantages. [4; 5; 6; 7]. The most perspective tomographic methods are based on nonlinear parameter. However, the use of currently available tomography schemes for formation of imaging medical systems is not always possible.

Attempts to implement various schemes of tomography have not led to the creation of optimal visualization systems [8]. Most of the current schemes of ultrasound tomography make use of linear dependence of amplitude increase of the second harmonic and waves of combination frequencies. These methods predominantly based on using the techniques of ultrasonic beam approximation [9]. The main disadvantages of such schemes are low resolving power and long time to perform researches. Parallel use of wave approach along with the radiation will improve the quality of the resulting image [10]. A resolution-limit of ultrasonic wave in linear tomography systems can range up to parts of a wave length, and a minimum of time for measuring is less than a second. However such systems feature high sophistication of hardware implementation and processing of obtained data. It is necessary to find new promising, from the viewpoint of practical implementation, quantitative evaluation methods of nonlinear parameter distribution pattern in biological mediums.

The paper presents results of stimulating the amplitude change of the higher harmonic of acoustical signal passing through the inclusions with the heterogeneous distribution of the nonlinear parameter.

An acoustical wave propagation process is described by three types of the component terms in the equation shown below: linear terms, nonlinear ones of the even powers and odd ones. To describe an acoustical wave field it is necessary to consider all the terms of harmonic expansion: the first term of the even power (of the squared nonlinearity) and the odd power (of the cubed nonlinearity). The following terms of expanding the even and odd powers can be considered as additive corrective pieces to the first terms of the corresponding nonlinearities.

Let us assume:

$$\varepsilon(r) \sim \frac{U_{2\omega}(r)}{U_{2\omega 0}(r)}, \quad (1)$$

$$\zeta(r) \sim \frac{U_{3\omega}(r)}{U_{3\omega 0}(r)}, \quad (2)$$

where ε(r) is the squared nonlinear parameter; ζ(r) is the cubed nonlinear parameter.

Let us consider the two cases of an acoustical wave propagation process: for one-dimensional wave in which the signal amplitude distribution is taken into account only in a wave line, and for a two-dimensional ultrasonic beam in which the changes were taken into account relating to the transverse distribution of amplitudes.

2. CASE OF THE ONE-DIMENSIONAL WAVE

An amplitude change will occur along Z-axis corresponding to a wave line in the one-dimensional wave case. To calculate amplitude of the one-dimensional signal we use nonlinear Earnshaw's equation written down in the form as follows [11; 12]:

$$\frac{\partial^3 \xi}{\partial t^2} = c_0^2 \frac{\partial^2 \xi / \partial x^2}{(1 + \partial \xi / \nabla x)^{\gamma+1}}, \quad (3)$$

where

$$\left(1 + \frac{\partial \xi}{\partial x}\right) = 1 - (\gamma+1)\frac{\partial \xi}{\partial x} + \frac{1}{2}(\gamma+1)(\gamma+2)\left(\frac{\partial \xi}{\partial x}\right)^2. \quad (4)$$

Then substitute equation (4) into the right part of equation (3), we'll obtain:

$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{c_0^2} \cdot \frac{\partial^2 \xi}{\partial x^2} = (\gamma+1)\frac{\partial \xi}{\partial x} \cdot \frac{\partial^2 \xi}{\partial x^2} - \frac{1}{2}(\gamma+1)(\gamma+2)\left(\frac{\partial \xi}{\partial x}\right)^2 \frac{\partial^2 \xi}{\partial x^2}, \quad (5)$$

where $(\gamma+1)\frac{\partial \xi}{\partial x} \cdot \frac{\partial^2 \xi}{\partial x^2}$ is the squared nonlinear term, $\frac{1}{2}(\gamma+1)(\lambda+2)\left(\frac{\partial \xi}{\partial x}\right)^2 \frac{\partial^2 \xi}{\partial x^2}$ is the cubed nonlinear term.

Using Khokhlov's method of a slowly changing profile, we will simplify equation (5) and deduce amplitude for the second and third harmonics.

For this purpose we will introduce a small parameter $\mu \sim (\gamma+1) \cdot A\frac{\omega}{c_0} \sim (\gamma+1) \cdot 2\pi A/\lambda \ll 1$ considering that displacement amplitude of particles of the medium should be small in comparison with the wave length.

Let us now turn from the initial coordinates x and t to the accompanying coordinates x1 and $\tau = t - x/c_0$, $\xi = F(\tau = t -_{x/c_0, x_1} = \mu x)$.

Then it will be:

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\partial^2 \xi}{\partial \tau^2},$$

$$\frac{\partial \xi}{\partial x} = -\frac{1}{c_0}\frac{\partial \xi}{\partial \tau} + \mu \frac{\partial \xi}{\partial x_1},$$

$$\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c_0^2}\frac{\partial^2 \xi}{\partial \tau^2} - \frac{2\mu}{c_0}\frac{\partial^2 \xi}{\partial \tau \partial x_1} + \mu^2 \frac{\partial^2 \xi}{\partial x_1^2}. \quad (6)$$

Having substituted the derivatives into equation (5) we'll obtain:

$$\mu^2 \frac{\partial^2 \xi}{\partial x_1^2} - \frac{2\mu}{c_0}\frac{\partial^2 \xi}{\partial \tau \partial x_1} + \frac{1}{c_0^2}\frac{\partial^2 \xi}{\partial \tau^2} = (\gamma+1)\left(\mu \frac{\partial \xi}{\partial x_1} - \frac{1}{c_0}\frac{\partial \xi}{\partial \tau}\right)\left(\mu^2 \frac{\partial^2 \xi}{\partial x_1^2} - \frac{2\mu}{c_0}\frac{\partial^2 \xi}{\partial \tau \partial x_1} + \frac{1}{c_0^2}\frac{\partial^2 \xi}{\partial \tau^2}\right). \quad (7)$$

If we neglect all the terms of μ^2 and μ^3 order we'll obtain:

$$-\frac{2\mu}{c_0}\frac{\partial^2 \xi}{\partial \tau \partial x_1} = (\gamma+1)\left(\frac{3\mu}{c_0^2}\frac{\partial^3 \xi}{\partial x_1 \partial \tau^2} - \frac{1}{c_0^3}\frac{\partial^3 \xi}{\partial \tau^3}\right) + \frac{1}{2}(\gamma+1)(\gamma+2)\left(-\frac{4\mu}{c_0^3}\frac{\partial^4 \xi}{\partial x_1 \partial \tau^3} - \frac{1}{c_0^4}\frac{\partial^4 \xi}{\partial \tau^4}\right)$$

here $\frac{\partial \xi}{\partial \tau} = \frac{\partial \xi}{\partial t} = u$ is the particle velocity, $x = \mu \cdot x_1$:

$$-\frac{2\mu}{c_0}\frac{\partial u}{\partial x} = (\gamma+1)\left(\frac{3}{c_0^2}u\frac{\partial u}{\partial x} - \frac{1}{c_0^3}\frac{\partial u}{\partial \tau}\right) + \frac{1}{2}(\gamma+1)(\gamma+2)\left(-\frac{4}{c_0^3}u^2\frac{\partial u}{\partial x} - \frac{1}{c_0^4}u^2\frac{\partial u}{\partial \tau}\right) \quad (8)$$

If it is assumed that $\varepsilon = (\gamma+1)/2$ is squared nonlinear parameter, and $\zeta = (\gamma+1) \cdot (\gamma+2)/6$ is cubed practical implementation of d nonlinear parameter, then we will obtain:

$$\frac{\partial u}{\partial x} = \frac{\varepsilon}{c_0^2}u\frac{\partial u}{\partial \tau} - \frac{\zeta}{c_0^3}u^2\frac{\partial u}{\partial \tau}. \quad (9)$$

Obtained equation (9) is simpler than equation (5), therefore, we will use it for separating out the solution of the second and third harmonics.

An equation of the travelling wave in the first approximation takes the form:

$$u(x,t) = A\sin\left(\omega\left(t - \frac{x}{c_0}\right)\right), \quad (10)$$

Using the method of successive approximations, we obtain the solution of equations for the second and third harmonics:

$$u_{2\omega} = \frac{\varepsilon \cdot \omega \cdot u_0^2 \cdot z}{2 \cdot c_0^2}\sin(2\omega\tau), \quad (11)$$

$$u_{3\omega} = \frac{\varepsilon \cdot \omega \cdot u_0^3 \cdot z}{2 \cdot c_0^3}\sin(3\omega\tau), \quad (12)$$

We considered the case when the one-dimensional acoustic wave had passed through the heterogeneous inclusion. Nonlinear parameter distribution in this inclusion changed according to the Gaussian law according to the equation:

$$\gamma(z) = \gamma_0 + \Delta\gamma 1 \exp - \frac{(z-a)^2}{b^2}, \quad (13)$$

where z is the wave propagation axis.

We assume that dependence of an amplitude change of the second harmonic, as the nonlinear parameter changes in heterogeneity, reflects the change of ratios $\frac{U_{2\omega}(z)}{U_{2\omega 0}(z)}$ and $\frac{U_{3\omega}(z)}{U_{3\omega 0}(z)}$ varying with the distance where:

$U_{2\omega}(z)$ is the amplitude of the second harmonic passed through biological tissue with heterogeneity;
$U_{2\omega 0}(z)$ is the amplitude of the second harmonic passed through biological tissue with the homogeneous distribution of the nonlinear parameter;
$U_{3\omega}(z)$ is the amplitude of the third harmonic passed through biological tissue with heterogeneity;
$U_{3\omega 0}(z)$ is the amplitude of the third harmonic passed through biological tissue with the homogeneous distribution of the nonlinear parameter.

To calculate the amplitude change of the second harmonic, we use equation (8), and we will solve this through $\varepsilon(z)$ for $U_{2\omega}(z)$ and through ε_0 for $U_{2\omega 0}(z)$.The solution to the equations take the forms:

$$U_{2\omega 0} = \frac{\varepsilon_0 \cdot \omega \cdot U_0^2 \cdot z}{2 \cdot c_0^2} \sin(2\omega\tau), \quad (14)$$

$$U_{2\omega} = \frac{\left(\varepsilon_0 + \int_0^i \varepsilon_z dz\right) \cdot \omega \cdot U_0^2 \cdot z}{2 \cdot c_0^2} \sin(2\omega\tau), \quad (15)$$

where $\varepsilon_0 = \frac{\gamma_0 + 1}{2}$, $\varepsilon_z = \frac{\gamma(z) + 1}{2}$,

$\gamma(z) = \gamma_0 + \Delta\gamma \cdot e^{-\frac{(z-a)^2}{b^2}}$,

i –Δγ.

The calculations are performed for the following parameters: $\omega/2\pi = 3.5$ MHz; $\Delta\gamma = 2, 7, 12$; $\gamma_0 = 6$; $a = 1.5$ mm.

The ratio $\frac{U_{2\omega}(z)}{U_{2\omega 0}(z)}$ is shown in figures 1 and 2 for various values of variations of the nonlinear parameter.

We use equation (9) to calculate the amplitude change of the third harmonic, and we will solve it by using $\varepsilon(z)$ for $U_{3\omega}(z)$ and ε_0 for $U_{3\omega 0}(z)$.
Then:

$$U_{3\omega 0} = \frac{\zeta_0 \cdot \omega \cdot U_0^3 \cdot z}{2 \cdot c_0^3} \sin(3\omega\tau), \quad (16)$$

$$U_{3\omega} = \frac{\left(\zeta_0 + \int_0^i \zeta_z dz\right) \cdot \omega \cdot U_0^3 \cdot z}{2 \cdot c_0^3} \sin(3\omega\tau) \quad (17)$$

where $\zeta_0 = \frac{(\gamma_0 + 1)(\gamma_0 + 2)}{6}$,

$\zeta(z) = \frac{(\gamma(z) + 1)(\gamma(z) + 2)}{6}$.

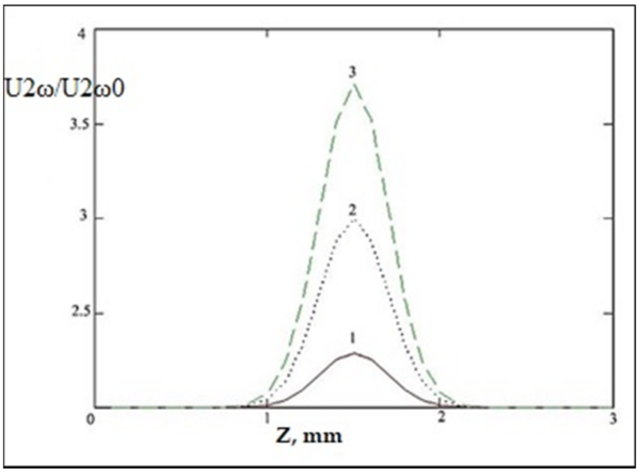

Figure 1. Amplitude change of the second harmonic when passing through heterogeneity at the Gaussian distribution of the nonlinear parameter for $\Delta\gamma = 2, 7, 12$ that correspond to curves 1, 2, and 3, respectively.

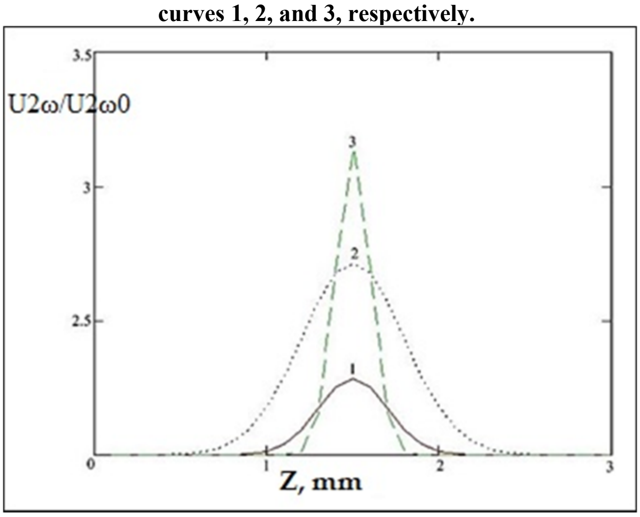

Figure 2. Amplitude change of the third harmonic when passing through heterogeneity at the Gaussian distribution of the nonlinear parameter for $\Delta\gamma = 2, 7, 12$; $a = 0.5, 1.5, 2$ that correspond to curves 1, 2, and 3, respectively.

The ratio $\frac{U_{3\omega}(z)}{U_{3\omega 0}(z)}$ is shown in figure 3 and 4 for various values of variations of the nonlinear parameter.

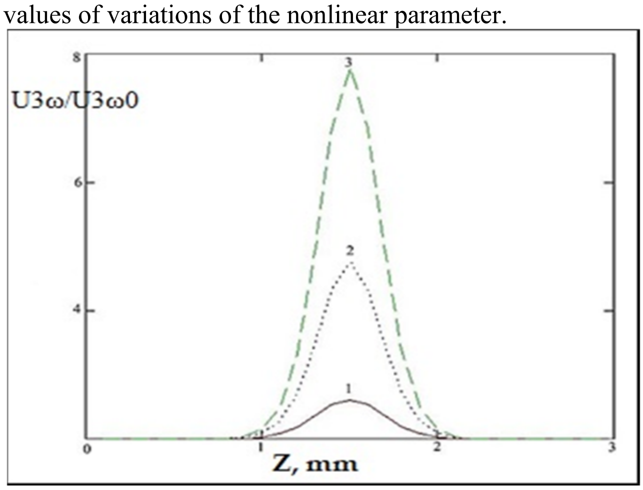

Figure 3. Amplitude change of the third harmonic when passing through heterogeneity at the Gaussian distribution of the nonlinear parameter for $\Delta\gamma = 2, 7, 12$ that correspond to curves 1, 2, and 3, respectively.

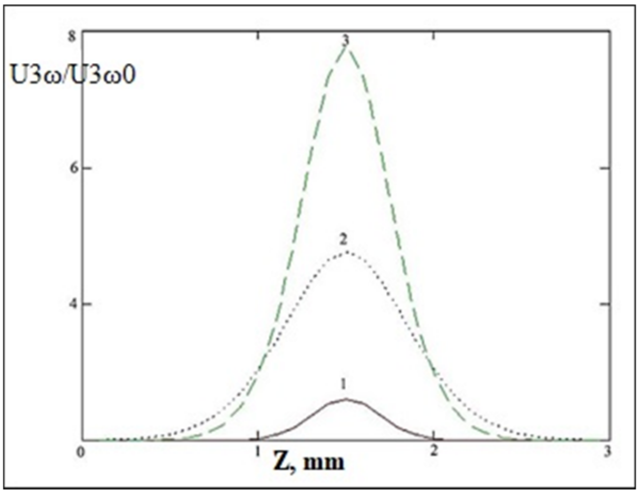

Figure 4. Amplitude change of the third harmonic when passing through heterogeneity at the Gaussian distribution of the nonlinear parameter for $\Delta\gamma = 2, 7, 12$; $a = 1, 2.5, 2$a = 1, 2.5, 2 that correspond to curves 1, 2, and 3, respectively.

The results of the performed calculations graphically shown in figure 1-4 demonstrated that at changing $\Delta\gamma$ by 10 one could see a change of the ratio amplitude of the second harmonic by 1.5, the ratio of the amplitude of third harmonic changed by 1.7; this points to the fact that the third harmonic is more sensitive to the change of the nonlinear parameter.

CASE OF THE TWO-DIMENSIAL ULTRASONIC BEAM

Geometry of the solvable problem for the case of a two-dimensional ultrasonic beam is shown in figure 5. Distribution of the nonlinear parameter was being considered with relation to the traverse X-coordinate taking into account the wave distance passed along Z-axis.

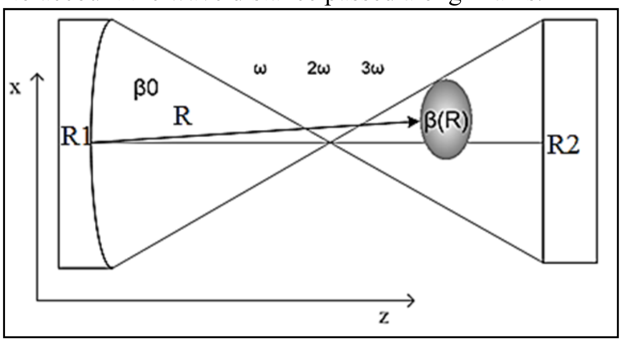

Figure 5. Geometry of the solvable problem.

In figure 5 R is the direction vector, R_1 is the radiator, R_2 is the receiver, β_0 is the value of the nonlinear parameter of adjoining tissues, $\beta(R)$ is the distribution of the nonlinear parameter in the heterogeneous inclusion.

We assume that heterogeneous inclusion has the ellipse-shaped form with the standard Gaussian distribution of the nonlinear parameter, then:

$$\gamma(r) = \gamma_0 + \Delta\gamma \exp\left[-\sum_j \frac{(x_j - X_j)}{a_j^2}\right], \quad (18)$$

where γ_0 is the nonlinear parameter of a homogeneous medium, $\Delta\gamma$ is the maximal variation of the nonlinear parameter of the heterogeneous inclusion from the homogeneous medium.

The propagation of the two-dimensional focused ultrasonic beam is considered by us in relation to the Khokhlov-Zabolotskaya-Kuznetsov equation (KZK) [13]:

$$\frac{\partial^2 p}{\partial\tau\partial z} - \frac{c}{2}\Delta_\perp p = \varepsilon(z)\frac{\partial^2 p^2}{\partial\tau^2} \quad (19)$$

For the second harmonic:

$$2i\omega\frac{\partial p^{2\omega}}{\partial z} - \frac{c}{2}\Delta_\perp p^{2\omega} = S_2(z, r_\perp), \quad (20)$$

where $s_2(z, r_\perp) = -4\omega^2 \cdot \varepsilon(r) \cdot p^\omega(z, r_\perp)$ is the function of the secondary radiation sources

$$\varepsilon(z) = \frac{\gamma(r) + 1}{2} \quad (21)$$

For the given problem:

$$s_2 = -4\omega^2 \cdot \varepsilon(r) \cdot p^\omega, \quad (22)$$

$$p^\omega(z = 0, r_\perp) = \sum_n A_n \exp\left(-\frac{B_n r_\perp}{a^2}\right). \quad (23)$$

As is the case with the one-dimensional acoustical wave, we deduce the distribution of the nonlinear parameter in heterogeneity from equation (1).

We do the same for $U_{2\omega 0}$ and $U_{2\omega}$ taking into account the transverse distribution.

The equation of amplitude change of the second harmonic of the wave passed through biological tissue with the homogeneous distribution of the nonlinear parameter is as follows:

$$U_{2\omega 0} = -2\pi\frac{\omega^2}{c_0}\varepsilon_0 z \sum_n A_n e^{-(1-iG)\cdot\frac{r_\perp^2}{a^2}}. \quad (24)$$

The equation of amplitude change of the second harmonic of the wave passed through the homogeneous biological tissue with the homogeneous distribution of the nonlinear parameter takes the form:

$$U_{2\omega} = U_{2\omega 0} + U_{2\omega}^{\varepsilon(r)}, \quad (25)$$

$$U_{2\omega}^{\varepsilon(r)} = -2\pi\frac{\omega^2}{c_0}\varepsilon(z) z \sum_n A_n e^{-(1-iG)\cdot\frac{r_\perp^2}{a^2}}. \quad (26)$$

The calculations are performed for the following parameters of the acoustic field and medium: $\omega/2\pi = 3.5$ MHz; $\Delta\gamma = 4$; $\gamma_0 = 6$; $x = 8$ mm.

The result of simulating the amplitude change of the second harmonic along X-axis is shown in figure 6.

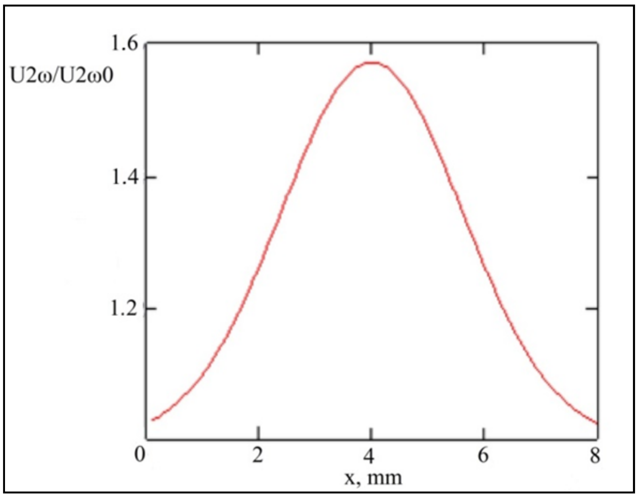

Figure 6. Amplitude change of the second harmonic of ultrasonic beam along transverse X-axis.

For the third harmonic we have:

$$2i\omega\frac{\partial p^{3\omega}}{\partial z}-\frac{c}{2}\Delta_{\perp}p^{3\omega}=S_3(z,r_{\perp}), \qquad (27)$$

where $s_3(z,r_{\perp})=-8\omega^3\cdot\zeta(r)\cdot p^{\omega}(z,r_{\perp})$ is the function of the third order sources:

$$\zeta(z)=\frac{(\gamma(r)+1)(\gamma(r)+2)}{6}. \qquad (28)$$

For the given problem we have:

$$s_3=-8\omega^3\cdot\zeta(r)\cdot p^{\omega}, \quad (29)$$

$$p^{\omega}(z=0,r_{\perp})=\sum\nolimits_n A_n\exp\left(-\frac{B_n r_{\perp}}{a^2}\right). \quad (30)$$

The amplitude change of the third harmonic of the wave passed through biological tissue with the homogeneous distribution of the nonlinear parameter takes the form:

$$U_{3\omega 0}=-2\pi\frac{\omega^3}{c_0^2}\zeta_0 z\sum\nolimits_n A_n e^{-(1-iG)\cdot\frac{r_{\perp}^2}{a^2}} \quad (31)$$

The amplitude change of the third harmonic of the wave passed through the heterogeneous biological tissue is as follows:

$$U_{3\omega}=U_{3\omega 0}+U_{3\omega}^{\zeta(r)} \quad (32)$$

$$U_{2\omega}^{\varepsilon(r)}=-2\pi\frac{\omega^2}{c_0}\varepsilon(z)z\sum\nolimits_n A_n e^{-(1-iG)\cdot\frac{r_{\perp}^2}{a^2}} \quad (33)$$

The calculations are performed for the following parameters of the acoustic field and medium: : $\omega/2\pi=3.5$ MHz; $\Delta\gamma=4$; $\gamma_0=6$; $x=8$ mm.

The result of simulating the amplitude change of the third harmonic along the transverse X-axis is presented in figure 7.

The obtained results of simulating the transverse distribution of the second harmonic when propagating the two-dimensional acoustic beam through biological tissue with heterogeneity allowed reproducing the distribution of the nonlinear parameter along the wave propagation line. The performed calculations demonstrated that the third harmonic was more sensitive to changes of the nonlinear parameter about by 2.2.

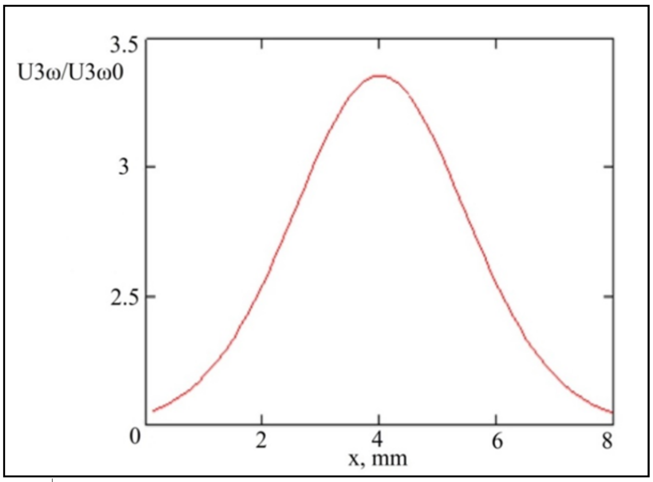

Figure 7. Amplitude change of the third harmonic of ultrasonic beam along transverse X-axis.

4. CONCLUSION

Simulating of the amplitude change of the higher harmonics of acoustical signal was performed when passing through biological tissues with the homogeneous distribution of the nonlinear parameter. The two cases of wave propagation were considered: one-dimensional wave (distribution of signal amplitude was taken into account only if it was in the direction of propagation) and two-dimensional ultrasonic beam; the changes were considered in relation to the transverse distribution of amplitude.

The obtained results of calculations allow reasoning that the distribution of the nonlinear parameter coincides with the amplitude change of the higher harmonics of acoustical signal, which are formed by nonlinear effects of acoustical wave interaction with biological medium. It has been found that the third harmonic level is low with regard to attenuation. Therefore, measurements of its parameters by experiment leads are problematic. High sensitivity of the third harmonic in comparison with the second harmonic will not give sufficient gain for designing system of imaging.

On the basis of the obtained regular relationships, it is possible to draw some conclusions:

- During the progress of the description of propagation of acoustical waves in biological tissues it is necessary to consider nonlinearities of the even power (a squared nonlinearity) and the first nonlinearity of an odd power (a cubed nonlinearity), as all the following terms of the even and odd orders of harmonic expansion can be considered as additional redetermining pieces to the corresponding first terms of the squared and cubed nonlinearities;
- The performed calculations make it possible to select a technique for solving a reverse problem that follows: finding the distribution of the nonlinear parameter in a homogeneous biological subject with subsequent imaging of its internal organ structures.

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